Journal of Engineering Physics and Thermophysics, Vol. 67, No. 1-2, 1994

# FRACTAL AND INFORMATION CHARACTERISTICS OF THE STRESSED STATE OF A MASSIF AND THEIR EFFECT ON THE PROCESS OF FRACTURE FORMATION

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UDC 622.24

*A probability-information method for describing the stressed state of a massif is developed. An analysis of the interrelationship of massif fracturing, the shape of the tool, and the stressed state of the massif in loading is made, and the dependence between the information characteristics of these distributions is found.*

A special feature of rocks that distinguishes them from artificial materials is the presence of numerous structural heterogeneities (blocks, fractures, grains, minerals) that have a significant effect on their destruction. Efforts to take account of these special features when describing the destruction process with current theoretical methods based on continuum mechanics and Griffith's theory lead to insurmountable computational difficulties.

In the present work we develop a mathematical model of rock destruction on the basis of information theory and percolation theory that enables us to take into account the presence of structural heterogeneities in rock.

Formation and growth of fractures in rocks occur through the emergence of connecting necks between microdefects in rock [1]. Thus, for a mathematical description of these processes flow theory is applicable [2]. Formation of a connecting neck between two microdefects is a random event, its randomness being due to both the heterogeneity of the properties of rock and the random component of the stress field. The stress field randomness is a consequence of numerous randomly oriented stress concentrators (structural heterogeneities of the massif) present in rock. Thus, the stressed state of the massif can be described by a probability density function of the stress intensity, and this function's parameters characterize the presence of structural heterogeneities in the massif. Below, the statistical entropy of this function (information of the stressed state) is chosen as a stressed state parameter. It is well known that there are systems of fractures in rock that form a block structure [1], and it exists at several levels (right down to grain sizes at the lower one). A growing fracture integrates existing fractures in rock simultaneously at many levels, i.e., goes along the block boundaries at different levels (scales). Thus, there is fractality of both the fracture and the stressed state that governs its trajectory, and the fractal dimensions of these objects depend on the massif structure and can also characterize this structure.

We deal with the process of fracture formation in the loading of rocks. The stress at a point of a massif may be considered a random variable since it is governed by the stress concentration on structural heterogeneities in rock, the stress superposition on each of them, and their size and orientation [1]. An information description is proposed in [3] for the shape of a tool that is based on a representation of the contact stress profile as a probability density; it is shown that the statistical entropy of this probability density characterizes the shape of the tool and is functionally connected with the energy intensity of rock destruction by this tool. We deal with the probability density of the stress intensity distribution $p(\sigma_i)$ as a function of size distribution of the stress concentrators in rock and the tool's shape. The function $p(\sigma_i)$ is related to the stress distribution over the massif by the formula

$$p(\sigma_i) \approx \frac{V}{\varepsilon^3} \sum_{V/\varepsilon^3} \delta\,[\sigma_i;\ \sigma_i(x,\ y,\ z)],$$

where $\varepsilon$ is the linear size of a conventional small element of the massif (the distance between microdefects) [4]; $\sigma_i(x, y, z)$ is the function that describes the stress distribution over the massif.

Institute of Superhard Materials, Academy of Sciences of the Ukraine, Kiev. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 67, Nos. 1-2, pp. 73-77, July-August, 1994. Original article submitted May 11, 1992.

Let $p(S)$ and $p(\sigma_{con})$ be the probability density of the areas of the cavity of existing stress concentrators and contact stresses on a loading site. Taking into account the direct proportionality between the contact stresses on the loading site and the stress intensity at massif points near a given point of the contact site [3] (this follows from smooth change in the contact stresses on the contact site and a solution to the Boussinesq problem) as well as the laws of stress distribution near a plane concentrator, we can write

$$\sigma_i = C\,\sigma_{con}\,S^{1/4}\,. \tag{1}$$

The distribution function $p(\sigma_i)$ is expressed in terms of $p(\sigma_{con})$ and $p(S)$ using the operation of convolution. We determine $p(S)$ and $p(\sigma_{con})$. In the course of surface deformation of rock at a constant load there is a redistribution of contact stresses (on account of surface crushing of the rock) that minimizes the stress entropy [5]. By writing the corresponding Lagrangian and minimizing it we obtain

$$p\,(\sigma_{con}) = \lambda_{con}\exp\,(-\,\lambda_{con}\,\sigma_{con})\,. \tag{2}$$

Formation of an infinite cluster in a given volume (which corresponds to formation of a fracture with a cavity area, equal to the area of this volume's diametral cross section $S$) occurs when the density of destroyed small elements (connecting necks between microdefects) attains the critical value $X_c$. This value is of the order of 0.2–0.4. Thus, the probability of formation of a fracture with the cavity area $S$ is determined as the probability of a given number of outcomes in sample with replacement:

$$p\,(S) = C_n^m\,r^m\,(1-r)^{n-m}\,, \tag{3}$$

$n = 0.095S^{2/3}\varepsilon^{-3}$, $m = X_c n$. At large $n$ and small $r$ the binomial distribution (3) reduces to the Poisson one:

$$p\,(S) = \psi\,(m\,;\,nr) = \frac{(nr)^m}{m\,!}\exp\,(-\,nr)\,. \tag{4}$$

The value of $\varepsilon$ is determined, as shown in [4], in terms of physical characteristics of the rock, and the value of $r$ is determined below.

The coefficient $C$ is a random variable and by applying to it the procedure of minimization of information [5] we find a distribution function $C$ similar to (2). We next take the logarithm of the function (1) and find the distribution function of the stress intensity logarithm as a convolution of the distributions (2) and (4) (with account for the formula of the relationship between the distribution function of a random variable and the distribution of its logarithm).

By representing (2) as a Poisson distribution and taking into account the stability of this distribution, after the corresponding rearrangements (which includes use of a convolution integral twice) we obtain

$$p\,(\sigma_i) = p_{non}p\,(\sigma_{con})\,p\,(S)\,p\,(C)\,, \tag{5}$$

where

$$p_{non} = 0.36\,\frac{nC_{av}}{\sigma_i}\left(\lambda_{con}\,\sigma_{con} + nr + \frac{C}{C_{av}}\right)^{m+1}\frac{(nr)^{-m}}{\lambda_{con}}\sum_{i=0}^{1} i\,(1-i)^n\,.$$

Here $p_{non}$ is a multiplier that takes account of nonlinear terms. The function (5) describes the stressed state occurring in a fractured massif in its loading with a tool of a prescribed shape. The coefficient $\lambda_{con}$ characterizes the tool shape and the loading energy, and the function (3) describes massif fracturing. The function (3) is derived from the conditions of the previous loading (with respect to that in question), which is characterized by the probability $r$.

We determine this probability. Let there be a loading of a massif whose conditions are also characterized by the functions (2) and (4) but with different coefficients $\lambda_{con}$ and $r$. Then the stressed state is characterized by the function $p_0(\sigma_i)$. Having specified the critical value of the stress intensity $\sigma_{cr}$ we obtain

$$r = \int_{\sigma_{cr}}^{\infty} p_0(\sigma_i)\, d\sigma_i . \tag{6}$$

Thus, there is a feedback that governs the formation of the fracture system with several loadings.

We next deal with the structure of the system of fractures itself and the stressed state that governs it. A fracture destroying the massif integrates existing fractures, and its trajectory between fractures of a given size is composed in turn of fractures of considerably smaller size, connecting necks between them, etc. Thus, fracture in rock is a fractal, and the same also refers to the stressed state governed by the size of a given fracture. Taking into account the formal definition of fractal dimension, we have

$$D = \ln\,[p(S_1)/p(S_2)]/\ln(S_2/S_1)\,, \tag{7}$$

where $D$ is the fractal dimension of the macrofracture; $S_1$ is a mathematical expectation of the fracture cavity area in the entire deformed volume; $S_2$ is the same in the volume with the diametral cross section $S_1$. If one calculates the fractal dimension in terms of other critical indices, then as a first approximation $D \approx 2.53$ [2]. The same fractal dimension obviously also characterizes the massif stressed state itself.

Expressions (2)-(7) imply a linear relationship between the information characteristics of the shape of the tool, the stressed state, and the fracturing of the massif. Using the Shannon formula to determine the information of the stressed state described by (5) with allowance made for the property of information additivity, we obtain

$$H_{ss} = H_{con} + H_c + H_s + \Delta H\,, \tag{8}$$

where $H_{ss}$ is the stressed-state information, $H_{ss} = \sum_{\sigma_i} p(\sigma_i) \ln p(\sigma_i)$; $H_{con}$ is the information of the contact stress distribution on the loading site; $H_s$ is the information of the size distribution of the stress concentrators in the massif; $H_c$ is the information of the stress concentrator distribution over the massif (distances to the point of loading and orientations); $\Delta H = \sum_{\sigma_i} p_{non} \ln p_{non}$. Thus, in massif deformation conversion of information occurs: the information of the shape of the tool and the structural heterogeneity distribution in the massif converts to stressed-state information. Further conversion is associated with conversion of the stressed-state information to that of the distribution of the formed fractures. It is evident that $r$ is related to $H_{ss}$ (the less the information of the distribution $p(\sigma_i)$, the larger the value of $r$). This relation depends on the form of the distribution $p(\sigma_i)$, and for the simplest case, namely, an exponent, it has the form

$$r = \exp\,[-\,\sigma_{cr} \exp(1 + H_{ss})]\,, \tag{9}$$

where $\sigma_{i_{av}}$ is the average value of the stress intensity in the massif.

The information of distribution (3) depends on $r$ by the following formula obtained with account for the Shannon formula:

$$H_s = n_{av}\,[r - X_c \ln(n_{av}\, r)] + \ln(X_c\, n_{av})\,!\,, \tag{10}$$

where $n_{av} = 0.095 S_1^{3/2} \varepsilon^{-3}$.

Thus, formulas (9) and (10) prescribe a reverse conversion of information: the stressed-state information converts to that of the fracture size distribution.

The information model developed above for the process of rock destruction can find wide practical application since it permits a much simpler formulation of many problems and hence simpler solution of them. For example, formulas (8) and (9) prescribe the relationship between the information of the contact stress distribution on the loading site and the massif destruction intensity. It follows that fragile rocks are destroyed most efficiently by spherical penetrators (a large contact site is ensured with little information of the contact stress distribution), plastic rocks by conical penetrators, and esily deformed rocks by cylindrical penetrators. This is confirmed by experiments of [6].

We compare simultaneous and successive loading of rock by two equal loads. In the first case the integrand in formula (6) is a convolution of two distributions (5) and in the second case it is their product. For the simplest case (the function (5) is an exponent) we obtain

$$r_1/r_2 = 1 + 0.5\,(1 - r_2)\,[1 - \ln(r_2/2)]\,,$$

where $r_1$ and $r_2$ correspond, respectively, to the simultaneous and successive loading. By rough estimates [3] $r_1/r_2 \sim 2.1$. Taking into account that $r$ is proportional to the newly formed fracture area, we infer that with loading simultaneity the distance between the penetrators at which chipping off occurs can be increased by a factor of 1.4. It is shown experimentally in [7] that this ratio (the width of the separated pillar in simultaneous and successive loading) is 1.33, i.e., 7% smaller. Thus, loading simultaneity (i.e., arrangement of bits in a group or in pairs) ensures an increase in rock destruction efficiency.

Fracture fractality affects the area of the newly formed surface in a massif, i.e., the energy intensity of destruction. By taking $r = X_c$ and taking account of the defect density as a function of the scale [2] we obtain

$$W = \frac{1}{d - D}\,\frac{\sigma_{cr} H_{ss}}{\ln X_c}\,,$$

where $d = 3$ is the dimension of the space.

Thus, a mathematical model of rock destruction based on methods of information theory and percolation theory is developed. A probability method for describing the stressed state of a massif is proposed. The massif stressed state as a function of rock fracturing and the shape of the tool is analyzed in a general form. It is shown that the process of rock destruction is a process of conversion of information.

The fractal dimension of the stressed state and the fractures formed is determined and its effect on the energy intensity of the destruction process is analyzed. A practical application of the model developed is shown; in particular, the appropriateness of arranging bits on bores in a group or in pairs is substantiated.

## NOTATION

$\sigma_i$, stress intensity at a point; $p(\sigma_i)$, probability of a given value of the stress intensity; $\sigma_{con}$, contact stress; $\delta$, Kronecker symbol; $V$, deformed volume; $\varepsilon$, distance between microdefects; $S$, fracture cavity area; $p(\sigma_{con})$, probability of a given value of the contact stresses; $C$, coefficient that takes into account the orientation of a fracture and its position relative to the loading site; $\lambda_{con}$, reciprocal of the average contact stress; $C_n^m$, binomial coefficient; $X_c$, critical density of destroyed elements; $n$, number of volumes of diameter $\varepsilon$ in a rock volume with the diametral cross section $S$; $\psi$, Poisson distribution; $r$, probability of formation of a connecting neck between two microdefects; $\sigma_{cr}$, ultimate strength of the rock; $C_{av}$, average value of $C$; $D$, fractal dimension; $H$, entropy; $W$, energy intensity of rock destruction.

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